System Control I Examination (2016-Fall semester) 2/6/2017 (3rd period)

(1) Answer the following questions for the system shown in Fig.1.

1. Calculate the transfer function $G_R(s) = \dfrac{Y(s)}{R(s)}$ when $D = 0$.
2. Calculate the transfer function $G_D(s) = \dfrac{Y(s)}{D(s)}$ when $R = 0$.

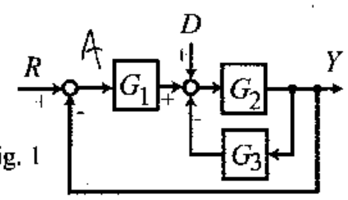

Fig. 1

(2) Answer the following questions, where the response $y(t)$ "must not" contain complex numbers.

1. Calculate the output response $y(t)$ for $G(s) = \dfrac{s-1}{(s+1)(s+2)}$, $Y = GU$ and $u(t) = 1$.
2. Calculate the output response $y(t)$ for $G(s) = \dfrac{1}{s(s^2+2s+3)}$, $Y = GU$ and $u(t) = \delta(t)$, where $\delta(t)$ is the unit impulse.

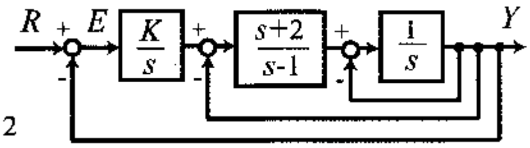

Fig. 2

(3) Answer the following questions for the system in Fig. 2.

1. Calculate the steady state error $e(\infty)$ when $r(t) = t$ $(t \geq 0)$.
2. Find the condition on $K(>0)$ where the closed loop system is stable.

(4) Draw Bode chart of $G(s) = \dfrac{10s}{(s+10)(s+0.1)}$ (only gain chart).

Exam.

Systems Control I (Answer)

1-1) $D=0$

$$G_L = G_1 \times \frac{G_2}{1+G_2G_3} \quad G_{cl} = \frac{Y}{R} = \frac{\frac{G_1G_2}{1+G_2G_3}}{1+\frac{G_1G_2}{1+G_2G_3}} = \frac{G_1G_2}{1+G_2G_3+G_1G_2} = G_R$$ 10

1-2)

$$A = R - Y \quad (R \to 0)$$

$$(AG_1 + D - YG_3)G_2 = Y \rightarrow -YG_1G_2 + DG_2 - YG_2G_3 = Y$$

$$Y(1+G_1G_2+G_2G_3) = DG_2$$ 10

$$G_D = \frac{Y}{D} = \frac{G_2}{1+G_1G_2+G_2G_3}$$

2-1) $u(t)=1 \xrightarrow{\mathcal{L}} U(s) = \frac{1}{s}$, $Y = \frac{s-1}{(s+1)(s+2)} \times \frac{1}{s} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$

$$A = \Big[Y \times s\Big]_{s=0} = \frac{-1}{1\times 2} = -\frac{1}{2}, \quad B = \Big[Y\times(s+1)\Big]_{s=-1} = \frac{-2}{1\times(-1)} = 2$$ 10

$$C = \Big[Y(s+2)\Big]_{s=-2} = \frac{-3}{-1\times(-2)} = -\frac{3}{2} \quad \text{thus } y(t) = -\frac{1}{2} + 2e^{-t} - \frac{3}{2}e^{-2t}$$

2-2) $u=\delta \rightarrow U=1$

$$Y = GU = \frac{1}{s(s^2+2s+3)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+3} \qquad A = \Big[Y\cdot s\Big]_{s=0} = \frac{1}{3}$$

$$p = -1 \pm \sqrt{1^2-3} = -1 \pm \sqrt{2}\,j \quad \text{complex number}$$ 20

$$Y = \frac{\frac{1}{3}(s^2+2s+3) + s\cdot Bs + Cs}{s(s^2+2s+3)} \Rightarrow \frac{1}{3} + B = 0 \quad B = -\frac{1}{3}$$

$$C + \frac{2}{3} = 0 \rightarrow C = -\frac{2}{3}$$

$$Y = \frac{1/3}{s} + \frac{-\frac{1}{3}s - \frac{2}{3}}{s^2+2s+3} = \frac{\underset{\cos}{\alpha(s+1)} + \underset{\sin}{\beta\sqrt{2}}}{(s+1)^2 + (\sqrt{2})^2} \rightarrow \alpha = -\frac{1}{3}, \quad \alpha + \sqrt{2}\beta = -\frac{2}{3}$$

$$\sqrt{2}\beta = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$$

Thus $y(t) = \frac{1}{3} + e^{-t}\left\{-\frac{\sqrt{2}}{6}\sin\sqrt{2}t - \frac{1}{3}\cos\sqrt{2}t\right\}$ $\quad \beta = -\frac{\sqrt{2}}{6}$

3-1) $\cancel{\frac{\frac{2}{s}}{1+\frac{2}{s}} = \frac{2}{s+2}}$, $\cancel{\frac{\frac{2(s+3)}{(s-1)(s+2)}}{1+\frac{2(s+3)}{(s-1)(s+2)}} = \frac{2(s+3)}{s^2+s-2+2s+6} = \frac{2s+6}{s^2+3s+4}}$ 50

$$\cancel{G_e = \frac{K(2s+6)}{s(s^2+3s+4)}, \quad E = \frac{R}{1+G_e}}$$

3-1) $\dfrac{\frac{1}{s}}{1+\frac{1}{s}} = \dfrac{1}{s+1} \rightarrow \dfrac{\frac{s+2}{(s-1)(s+1)}}{1+\frac{s+2}{(s-1)(s+1)}} = \dfrac{s+2}{s^2+s+1}$

$G_e = \dfrac{K(s+2)}{s(s^2+s+1)}$, $\underset{\substack{\uparrow \\ t}}{\mathcal{L}[r(t)]} = \dfrac{1}{s^2}$, $E = \dfrac{R}{1+G_e} = \dfrac{\frac{1}{s^2}}{1+\frac{K(s+2)}{s^3+s^2+s}}$

steady state error

$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \dfrac{s^2+s+1}{s^3+s^2+(1+K)s+2K} = \dfrac{1}{2K}$ 15 $= \dfrac{\frac{1}{s^2}(s^3+s^2+s)}{s^3+s^2+(1+K)s+2K}$

3-2)

$G_{ce} = \dfrac{G_e}{1+G_e} = \dfrac{*}{1+\frac{K(s+2)}{s^3+s^2+s}} = \dfrac{\phi}{s^3+s^2+(1+K)s+2K} \Rightarrow D(s)$

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Routh Method

s^3	1	$1+K$
s^2	1	$2K$
s^1	A	
s^0	B	

$A = -\frac{1}{1}\begin{vmatrix} 1 & 1+K \\ 1 & 2K \end{vmatrix} = (1+K) - 2K > 0 \rightarrow 1 > K$

$B = -\frac{1}{A}\begin{vmatrix} 1 & 2K \\ A & 0 \end{vmatrix} = 2K > 0$ totally $0 < K < 1$

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~~3-2)~~

4) $G(s) = \dfrac{10s}{(s+10)(s+0.1)} = \dfrac{0.1\times 10\times 10s}{(0.1s+1)(10s+1)} = \underset{A}{\dfrac{1}{0.1s+1}} \times \underset{B}{\dfrac{1}{10s+1}} \times \underset{C'}{s} \times \underset{D}{10}$

$\omega T = 1 \rightarrow \omega = \frac{1}{T} \Rightarrow \frac{1}{0.1} = 10$ (A) $\underline{\underline{Ts+1}}$

$\Rightarrow \frac{1}{10} = 0.1$ (B)

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