Answer Sheet of No. 1 (Examination of Systems Control I)

27th, Jan. 2020

Name

(1) ① at point A : $0 = -k_2(y-x) + u$

B : $m\ddot{x} = -k_1 x - d\dot{x} + k_2(y-x)$

② From A → $y - x = \frac{u}{k_2} \Rightarrow x = y - \frac{u}{k_2}$, $\dot{x} = \dot{y} - \frac{\dot{u}}{k_2}$, $\ddot{x} = \ddot{y} - \frac{\ddot{u}}{k_2}$ substitute

$$m(\ddot{y} - \frac{\ddot{u}}{k_2}) = -k_1(y - \frac{u}{k_2}) - d(\dot{y} - \frac{\dot{u}}{k_2}) + u$$

$$m\ddot{y} + d\dot{y} + k_1 y = \frac{m}{k_2}\ddot{u} + \frac{d}{k_2}\dot{u} + (\frac{k_1}{k_2} + 1)u$$

$$k_2 m\ddot{y} + k_2 d\dot{y} + k_1 k_2 y = m\ddot{u} + d\dot{u} + (k_1+k_2)u \quad \left(\frac{k_1+k_2}{k_2}\right)$$

③ $\downarrow \mathcal{L}$

$$(k_2 m s^2 + k_2 d s + k_1 k_2)Y = (ms^2 + ds + (k_1+k_2))U$$

$$G = \frac{Y}{U} = \frac{ms^2 + ds + (k_1+k_2)}{k_2 m s^2 + k_2 d s + k_1 k_2}$$

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(2) ① Calculate $G_a = \frac{Y}{U}$, $\{K(U-Y) + 2sU\} \times \frac{1}{s(s+1)} = Y$

$$Y\left(1 + \frac{K}{s(s+1)}\right) = \frac{(K+2s)}{s(s+1)}U \Rightarrow G_a = \frac{Y}{U} = \frac{K+2s}{s^2+s+K}$$

$$\frac{s^2+s+K}{s(s+1)}$$

② $U = \frac{1}{s}$, $K = 1$

$$Y = \frac{2s+1}{s^2+s+1} \times \frac{1}{s} = \frac{A(s+\frac{1}{2}) + B \times \frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} + \frac{C}{s} = \frac{As(s+\frac{1}{2}) + Bs\cdot\frac{\sqrt{3}}{2}}{s(s^2+s+1)} + \frac{1 \times (s^2+s+1)}{}$$

$C = [GU \times s]_{s=0} = \frac{1}{1} = 1$,

s^2 : $A + 1 = 0 \to A = -1$

s^1 : $-\frac{1}{2} + \frac{\sqrt{3}}{2}B + 1 = 2$

$\frac{\sqrt{3}}{2}B = 2 - 1 + \frac{1}{2} = \frac{3}{2} \to B = \frac{3}{\sqrt{3}} = \sqrt{3}$

Thus $y(t) = 1 + e^{-\frac{t}{2}}\left(\sqrt{3}\sin\frac{\sqrt{3}}{2}t - \cos\frac{\sqrt{3}}{2}t\right)$

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(3) ①

$$G_{cl} = \frac{\frac{5k}{s(s+2)}}{1+\frac{5k}{s(s+1)(s+2)}} = \frac{5k(s+1)}{s^3+3s^2+2s+5k}$$

② $D = s^3+3s^2+2s+5k \rightarrow$ Routh method

s^3	1	2
s^2	3	$5k$
s^1	A	
s^0	B	

$A = -\frac{1}{3}\begin{vmatrix} 1 & 2 \\ 3 & 5k \end{vmatrix} = (6-5k) > 0 \qquad k < \frac{6}{5}$

$B = -\frac{3}{A}\begin{vmatrix} 3 & 5k \\ A & 0 \end{vmatrix} = k > 0 \qquad \therefore\ 0 < k < \frac{6}{5}$

③ $E = \frac{R}{1+G_l} = \frac{1/s^2}{1+\frac{5k}{s^3+3s^2+2s}} = \frac{\frac{1}{s^2}(s^3+3s^2+2s)}{s^3+3s^2+2s+5k}$

$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s^2+3s+2}{s^3+3s^2+2s+5k} = \frac{2}{5k} < 1 \Rightarrow 2 < 5k \Rightarrow k > \frac{2}{5}$

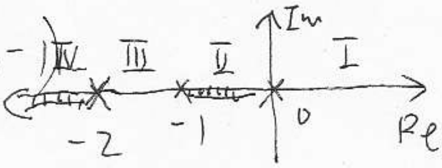
Thus totally $\frac{2}{5} < k < \frac{6}{5}$

(4) ① G_{cl} for Fig 4 $= \frac{\frac{KN}{D}}{1+\frac{KN}{D}} = \frac{KN}{D+KN} \rightarrow D+KN = s^3+3s^2+2s+5k$

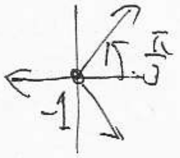
Thus $D = s^3+3s^2+2s$

$n=3, m=0 \qquad N=5$

② $G_p = \frac{N}{D} = \frac{5}{s(s+2)(s+1)}$ $(p = 0, -2, -1)$



$\theta = \frac{(2k+1)}{3}\pi = \frac{\pi}{3}, \pi, \frac{5}{3}\pi$



$\alpha = \frac{\sum p - \sum z}{3} = \frac{0-2-1}{3} = -1$

branch point $\Rightarrow \frac{d}{ds}\left(\frac{1}{G_p}\right) = \frac{d}{ds}(s^3+3s^2+2s) = 3s^2+6s+2 \Rightarrow s = \frac{-6\pm\sqrt{36-24}}{6} = \frac{-6\pm 2\sqrt{3}}{6}$

$= -1 \pm \frac{\sqrt{3}}{3}$

$\simeq -1.5$ or -0.5

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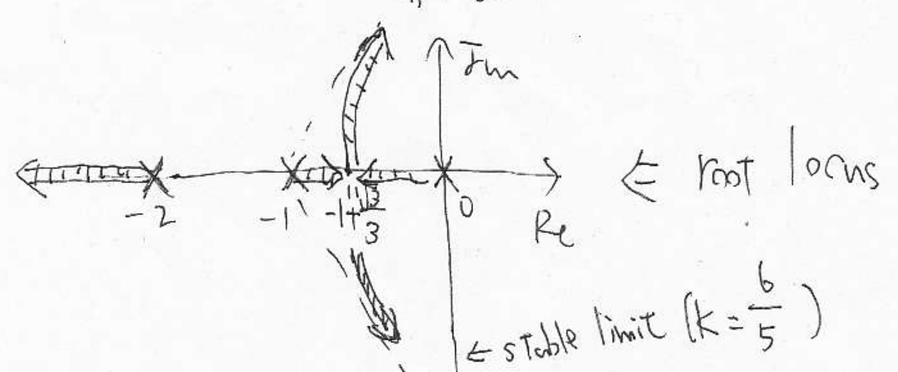

⇐ root locus

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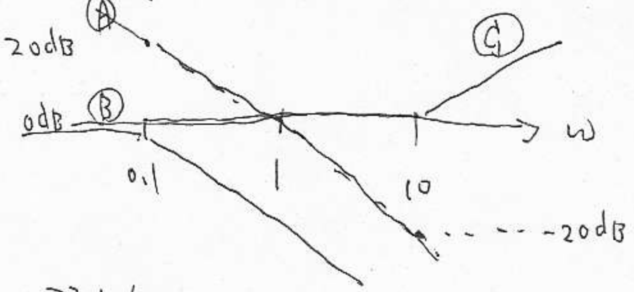
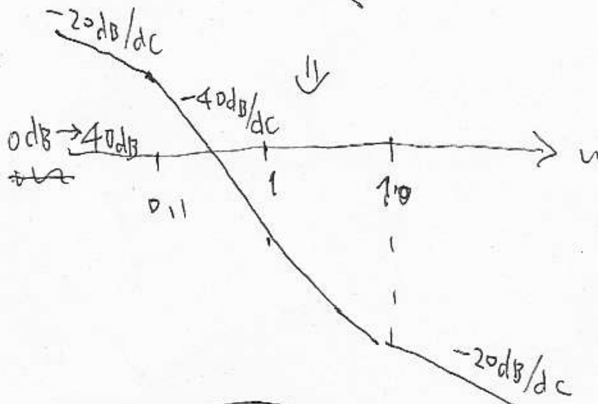
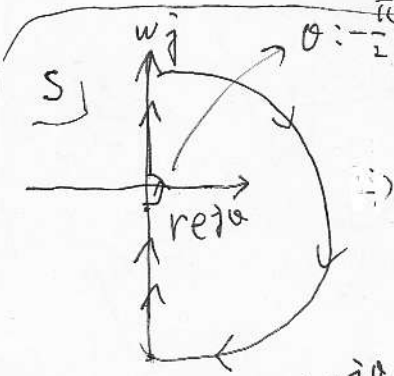
(5) ① $G = \dfrac{S+10}{s(S+0.1)} = \dfrac{1}{s} \times \dfrac{10}{10s+1} \times \dfrac{0.1s+1}{0.1} = \underset{Ⓐ}{\dfrac{1}{s}} \times \underset{Ⓑ}{\dfrac{1}{10s+1}} \times \underset{Ⓒ}{(0.1s+1)} \times 10^2$

Ⓑ $\omega = 0.1$ Ⓒ $\omega = 10$



⇓





$\theta : -\frac{\pi}{2} \to 0 \to \frac{\pi}{2}$ $G_e(jw)^2$

$\boxed{-1 = e^{j\pi}}$

② $G_e = \dfrac{2S+1}{S(S-1)} = \dfrac{2S+1}{S^2-S}$ → $\boxed{P = 1}$

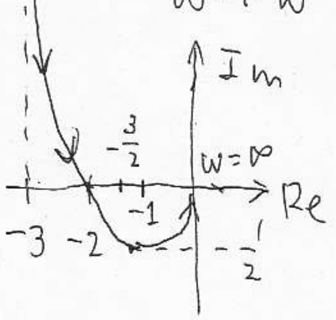
$G_e(jw) = \dfrac{2wj+1}{-w^2+wj} = \dfrac{(2wj+1)(-w^2-wj)}{w^4+w^2}$

$= \dfrac{-3w^2+(w-2w^3)j}{w^4+w^2}$

$X = \dfrac{-3}{w^2+1}$

$w=0$ $Y = \dfrac{1-2w^2}{w^3+w}$

w	X	Y
0	-3 (faster)	$\frac{1}{0} = +\infty$
$0.7 \simeq \frac{\sqrt{2}}{2}$	-2	0
1	$-\frac{3}{2}$	$-\frac{1}{2}$
∞	$\frac{-3}{\infty^2} = -0$ (faster)	$\frac{-2}{\infty} = -0$



vector locus

$G_e(re^{j\theta}) = \dfrac{2re^{j\theta}+1}{r^2e^{2j\theta} - re^{j\theta}}$

when $r \to 0 \simeq \dfrac{+1}{\ominus r \cdot e^{j\theta}}$

$= \dfrac{1}{r} \times e^{-j\theta} \times e^{\pi j}$

$= \dfrac{1}{r} e^{(\pi - \theta)j}$

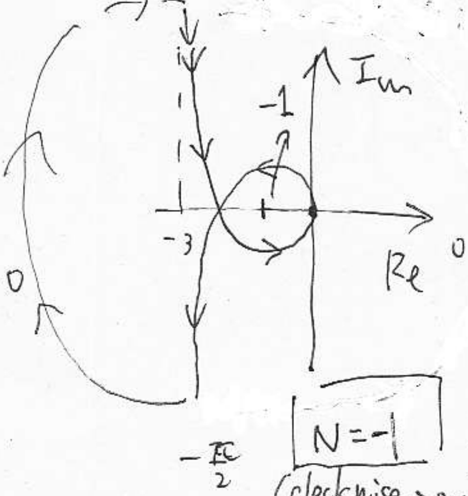
$\theta: -\frac{\pi}{2} \to \pi - \theta = \frac{3}{2}\pi = -\frac{\pi}{2}$

$\theta = 0 \to \pi - \theta = \pi$

$\theta = \frac{\pi}{2} \to \pi - \theta = \frac{\pi}{2}$

G_e

Nyquist locus



$-\frac{\pi}{2}$

$\boxed{N = -1}$

(clockwise ⇒ positive)

$Z = N + P = 0$ (stable!)