Systems Control I Exam. (2018-Fall Semester) 2/4/2019 (2nd period, 10:30-12:00)

(1) Answer the following questions.

1. Calculate $F(s) = \mathcal{L}[f(t)]$ for the function $f(t)$ in Fig.1 where $f(t) = 1 - t \ (0 \leq t \leq 2), \ \ f(t) = 0 \ (t > 2)$.
2. Shwo the original differential equation on $y(t)$ including $u(t)$ for the system in Fig. 2. ($u(t) = \mathcal{L}^{-1}[U(s)]$, $y(t) = \mathcal{L}^{-1}[Y(s)]$).
3. Calculate the time response $y(t)$ when $u(t) = 1 \ (t \leq 0), \ \ u(t) = 0 \ (t < 0)$, for the system in Fig. 2.

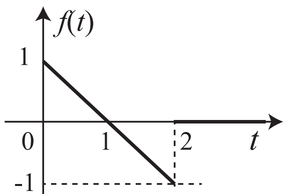


Fig. 1

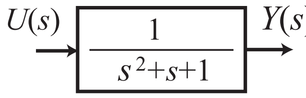


Fig. 2

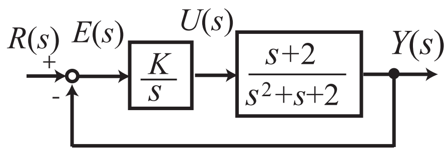


Fig. 3

(2) For the system in Fig. 3, answer the following questions.

1. Show the closed loop transfer function $G_{cl}(s) = \dfrac{Y}{R}$ of the system in Fig. 3, where K is a feedback gain.
2. Show the condition on K for the stable G_{cl}.
3. Show the condition on K where the steady state error (stady state velocity error) is $e(\infty) < 2$, when $r(t) = t \ \ (t \geq 0)$.
4. Draw a rough sktch of the root locus.

(3) Answer the following questions.

1. Draw the Bode chart for the system of $G(s) = \dfrac{s + 10}{s(5s + 1)}$ only for the gain chart by polygonal approximation using the given semi-log scale graph paper.
2. Drawing the vector locus of $G(s) = \dfrac{2s + 1}{s(s - 1)}$.

Model Answers for Systems Control I Exam. (2018 Fall) 2019/2/14

(1)

(1-1) $F(s) = \int_0^\infty e^{-st} f(t)\,dt = \int_0^2 (1-t)e^{-st}\,dt = \int_0^2 e^{-st}\,dt - \int_0^2 t e^{-st}\,dt$ (A)

$f g' = -f' g + [fg]$

$= -\frac{1}{s}\left[e^{-st}\right]_0^2 - \left\{ \left[t \times \left(\frac{-1}{s}\right) e^{-st}\right]_0^2 + \frac{1}{s}\int_0^2 e^{-st}\,dt \right\}$ $-\frac{1}{s}$(A) +(A)

$\left(1 - \frac{1}{s}\right)\left(\frac{1-e^{-2s}}{s}\right) - \frac{-2}{s}e^{-2s}$ (A)

$= \frac{e^{-2s}+1}{s} + \frac{e^{-2s}-1}{s^2}$ (A)

(1-2) $Y = \frac{1}{s^2+s+1}U$

$\mathcal{L}^{-1}$ $(s^2+s+1)Y = U$

$\ddot{y} + \dot{y} + y = u$

(1-3) $u(t) = 1 \rightarrow U(s) = \frac{1}{s}$, $Y = \frac{1}{s(s^2+s+1)} = \frac{A}{s} + \frac{B(s+\frac{1}{2})}{(s+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} + \frac{C \times \frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$

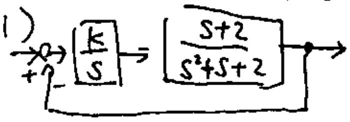
$A = [Y \cdot s]_{s=0} = 1$, $Y = \frac{(s^2+s+1)\times 1 + Bs(s+\frac{1}{2}) + Cs\frac{\sqrt{3}}{2}}{s(s^2+s+1)} = \frac{1}{\vdots}$

on $s^2 \rightarrow$

$B + 1 = 0 \rightarrow B = -1$, on $s \rightarrow 1 + \frac{-1}{2} + \frac{\sqrt{3}}{2}C = 0$, $\frac{\sqrt{3}}{2}C = -\frac{1}{2} \rightarrow C = -\frac{1}{2} \times \frac{2}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ (B)

Thus $y(t) = 1 - e^{-\frac{t}{2}}\left\{\cos\frac{\sqrt{3}}{2}t + \frac{\sqrt{3}}{3}\sin\frac{\sqrt{3}}{2}t\right\}$

(2) (2-1)



$G_{cl} = \frac{\vdots}{1 + \frac{K(s+2)}{s^3+s^2+2s}} = \frac{K(s+2)}{s^3+s^2+(2+K)s+2K}$

(2-2) $D(s) = s^3 + s^2 + (2+K)s + 2K$

Routh table

s^3	1	$2+K$
s^2	1	$2K$
s^1	A	
s^0	B	

$A = -\frac{1}{1}\begin{vmatrix} 1 & 2+K \\ 1 & 2K \end{vmatrix} = (2+K) - 2K > 0 \rightarrow 2 > K$

$B = -\frac{1}{A}\begin{vmatrix} 1 & 2K \\ A & 0 \end{vmatrix} = 2K > 0$

$0 < K < 2$

(2-3) $e(\infty) < 2$, $E = \frac{R}{1+G_e} = \frac{\frac{1}{s^2}}{1 + \frac{K(s+2)}{s^3+s^2+2s}} = \frac{\frac{1}{s^2}(s^3+s^2+2s)}{s^3+s^2+(2+K)s+2K} = \frac{\frac{1}{s}(s^2+s+2)}{\vdots}$

$r(t) = t \rightarrow R = \frac{1}{s^2}$

$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s^2+s+2}{s^3+s^2+(2+K)s+2K} = \frac{2}{2K} < 2 \rightarrow 1 < 2K$

$K > \frac{1}{2}$

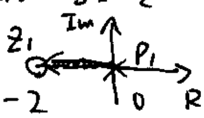
Thus totally $\frac{1}{2} < K < 2$

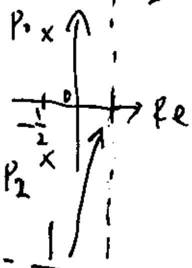
(2-4) root locus

$G_p = \frac{s+2}{s(s^2+s+2)}$

poles: $s = 0$, $\frac{-1 \pm \sqrt{1-8}}{2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}j$

zero: $s = -2$

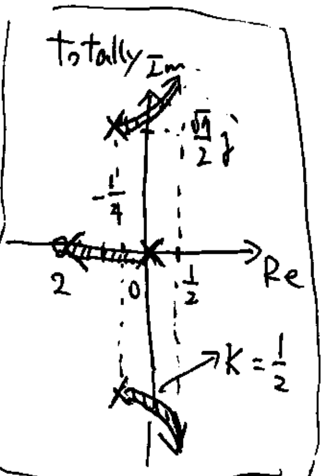




asymptote

$\alpha = \frac{\sum P - \sum Z}{3-1} = \frac{-1+2}{2} = \frac{1}{2}$

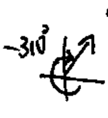
$\theta = \frac{2k+1}{3-1}\pi = \frac{\pi}{2}, \frac{3}{2}\pi$



Starting angle from $\left(-\frac{1}{2} + \frac{\sqrt{7}}{2}j\right)$ $\phi_0 = \psi - \phi - 180^\circ \simeq 70^\circ - (110^\circ + 90^\circ) - 180^\circ \simeq -310^\circ$

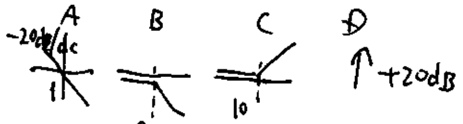
$\psi = \angle(\vec{P}_0 - \vec{Z}_1)$, $\phi = \angle(\vec{P}_0 - \vec{P}_1) + \angle(\vec{P}_0 - \vec{P}_2)$

$\rightarrow \simeq 70^\circ$ $\quad \simeq 110^\circ + 90^\circ$



(3-1) Bode Chart

$G = \frac{s+10}{s(5s+1)} = \frac{1}{s} \times \frac{1}{5s+1} \times (0.1s+1) \times 10$



$\Rightarrow$ see another sheet

(3-2) $G = \frac{2s+1}{s(s-1)} = \frac{s(s+1)(2s+1)}{s^2(s^2-1)} \overset{s=j\omega}{\Rightarrow} \frac{(-2\omega^3+\omega)j - 3\omega^2}{\omega^2(\omega^2+1)}$

$X = \frac{-3}{\omega^2+1}$, $Y = \frac{1-2\omega^2}{\omega(\omega^2+1)}$

ω	X	Y
0	−3	$\frac{1}{0} = \infty$
$\frac{1}{\sqrt{2}}$	−2	0
1	$-3/2$	$-1/2$
∞	-0	-0

(X is faster)