24th. Oct., 2019

(70)

Assignment (Robotics I, Robotics) ~~2nd~~ 1st

(M2, M1, B4, B3, others: CIRCLE ONE)   Major (Department)

Name(KANJI or English) Model Answer

[1] Prove $\underbrace{({}^AR_B)^{-1} = {}^BR_A}_{(1\text{-}1)}$, $\underbrace{({}^AR_B)^{-1} = ({}^AR_B)^T}_{(1\text{-}2)}$, $\underbrace{{}^AR_C = {}^AR_B\,{}^BR_C}_{(1\text{-}3)}$

(1-1) ${}^A\mathbf{r} = {}^AR_B\,{}^B\mathbf{r}$, ${}^B\mathbf{r} = {}^BR_A\,{}^A\mathbf{r}$ $\Rightarrow$ thus ${}^A\mathbf{r} = {}^AR_B\,{}^BR_A\,{}^A\mathbf{r} \Rightarrow {}^AR_B\,{}^BR_A = I$

$\Downarrow$

${}^BR_A = ({}^AR_B)^{-1}$

(1-2) $$({}^AR_B)^T({}^AR_B) = \begin{bmatrix} {}^A\mathbf{x}_B^T \\ {}^A\mathbf{y}_B^T \\ {}^A\mathbf{z}_B^T \end{bmatrix} \left[{}^A\mathbf{x}_B \,\vdots\, {}^A\mathbf{y}_B \,\vdots\, {}^A\mathbf{z}_B \right] = \begin{bmatrix} {}^A\mathbf{x}_B \cdot {}^A\mathbf{x}_B & {}^A\mathbf{x}_B \cdot {}^A\mathbf{y}_B & {}^A\mathbf{x}_B \cdot {}^A\mathbf{z}_B \\ {}^A\mathbf{y}_B \cdot {}^A\mathbf{x}_B & {}^A\mathbf{y}_B \cdot {}^A\mathbf{y}_B & {}^A\mathbf{y}_B \cdot {}^A\mathbf{z}_B \\ {}^A\mathbf{z}_B \cdot {}^A\mathbf{x}_B & {}^A\mathbf{z}_B \cdot {}^A\mathbf{y}_B & {}^A\mathbf{z}_B \cdot {}^A\mathbf{z}_B \end{bmatrix}$$

$(\mathbf{x}^T\mathbf{x} = \mathbf{x} \cdot \mathbf{x})$

using $|{}^A\mathbf{x}_B|^2 = {}^A\mathbf{x}_B \cdot {}^A\mathbf{x}_B = 1$

${}^A\mathbf{x}_B \cdot {}^A\mathbf{y}_B = |{}^A\mathbf{x}_B||{}^A\mathbf{y}_B|\cos\theta = 0$ . . .

$$\Rightarrow \; = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus $({}^AR_B)^T = ({}^AR_B)^{-1}$

(1-3) ${}^A\mathbf{r} = {}^AR_C\,{}^C\mathbf{r}$, ${}^B\mathbf{r} = {}^BR_C\,{}^C\mathbf{r}$

$= {}^AR_B\,{}^B\mathbf{r} = {}^AR_B\,{}^BR_C\,{}^C\mathbf{r}$

then ${}^AR_C = {}^AR_B\,{}^BR_C$

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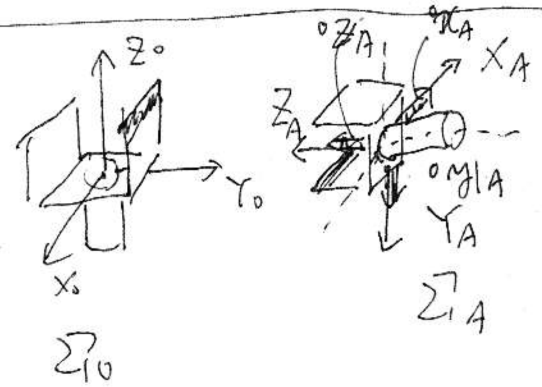
[2]



$${}^0R_A = \left[{}^0\mathbf{x}_A \;\; {}^0\mathbf{y}_A \;\; {}^0\mathbf{z}_B \right] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$