(1) Original orientation of a hand is given in Fig.1 (A). After some rotations, we have an another

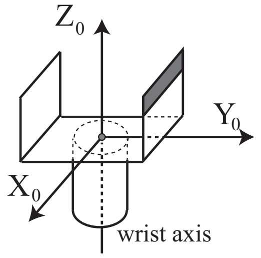


Fig. 1 (A)

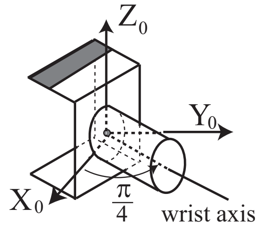


Fig. 1 (B)

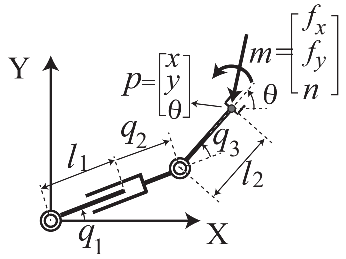


Fig. 2

orientation of the hand shown in Fig.1 (B).
(1-1) Find the rotation matrix ${}^0R$ which represents the orientation of the hand in Fig.1 (B).
(1-2) Find Euler parameters (ϕ, θ, ψ) which rotate the hand in Fig.1 (A) to the hand in Fig.1 (B).

(2) In Fig.2, we have a horizontal planer (X-Y) 3-DOF robotic arm, where q_1, q_2, q_3 are joint variables.
(2-1) Describe the point $\boldsymbol{p} = [x, y, \theta]^T$ (hand part of the arm) as a function of q_1, q_2, q_3.
(2-2) Show the velocity $\dot{\boldsymbol{p}}$ of the point $\boldsymbol{p}$ using Jacobian matrix $J(\boldsymbol{q})$. Show also each element of the $J(\boldsymbol{q})$.
(2-3) When a force and moment vector $\boldsymbol{m} = [f_x, f_y, n]^T$ is added at point $\boldsymbol{p}$, calculate the holding torque $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^T$ for each joint q_1, q_2, q_3. Note that the τ_3 is joint force not torque actually.

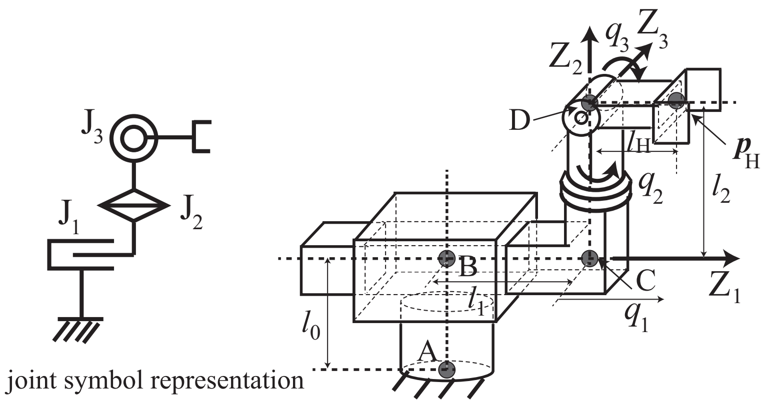


Fig. 3

(3) Answer the following questions on Fig. 3.
(3-1) Show the geometrical relationship of the coordinate frame Σ_0, Σ_1, Σ_2, and Σ_3 including the points A $\sim$ D. Follow the recommendations $Z_0 \equiv Z_1$, $X_0 \equiv X_1$, $X_3 \equiv X_2$.
(3-2) Find the Denaviet-Hartenberg parameters for the robot. Note that the figure is drawn for the case of $q_1 = q_2 = q_3 = 0$ case.
(3-3) How do you represents the hand point ${}^0\boldsymbol{p}_H$ in Σ_0 using homogeneous transformation matrix ${}^0T_3$. Where you do not need to show the actual elements of ${}^0T_3$.

Model Answer

Nov. 26th, 2019

(65)

Answer Sheet (Robotics Mid-Exam and Robotics I Exam) No. /

(M2, M1, B4, B3, others: CIRCLE ONE) Major (Department)

Name(KANJI or English) Student ID:

[1] (1-1) ${}^0x = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$, ${}^0y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, ${}^0z = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$ thus ${}^0R = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{bmatrix}$

(1-2) $R_{13} = -\frac{\sqrt{2}}{2}$, $R_{23} = -\frac{\sqrt{2}}{2}$, $R_{33} = 0$, $R_{32} = 1$, $R_{31} = 0$

$\theta = \mathrm{atan2}\left(\pm\sqrt{\tfrac{1}{2}+\tfrac{1}{2}}, 0\right) = +90°, -90°$, $S_\theta = 1$ $(90°)$ or -1 $(-90°)$

$S_\theta = 1 \rightarrow \phi = \mathrm{atan2}\left(\frac{-\sqrt{2}/2}{1}, \frac{-\sqrt{2}/2}{1}\right) = -135°$, $\psi = \mathrm{atan2}\left(\frac{1}{1}, -0\right) = 90°$

$(\theta = 90°)$

$S_\theta = -1 \rightarrow \phi = \mathrm{atan2}\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = 45°$, $\psi = \mathrm{atan2}\left(-\frac{1}{1}, 0\right) = -90°$

$(\theta = -90°)$ Thus $(\phi, \theta, \psi) = (-135°, 90°, 90°)$ or $(45°, -90°, -90°)$

$\frac{5}{4}\pi \leftarrow \left(-\frac{3}{4}\pi, \frac{\pi}{2}, \frac{\pi}{2}\right)$ or $\left(\frac{\pi}{4}, -\frac{\pi}{2}, -\frac{\pi}{2}\right)$

[2] (2-1) ${}^1p = \begin{bmatrix} X \\ Y \\ \theta \end{bmatrix} = \begin{bmatrix} (l_1+q_2)\cos q_1 + l_2\cos(q_1+q_3) \\ (l_1+q_2)\sin q_1 + l_2\sin(q_1+q_3) \\ q_1+q_3 \end{bmatrix}$, ${}^1\dot{p} = \begin{bmatrix} \frac{\partial X}{\partial q_1} & \frac{\partial X}{\partial q_2} & \frac{\partial X}{\partial q_3} \\ \frac{\partial Y}{\partial q_1} & \frac{\partial Y}{\partial q_2} & \frac{\partial Y}{\partial q_3} \\ \frac{\partial \theta}{\partial q_1} & \frac{\partial \theta}{\partial q_2} & \frac{\partial \theta}{\partial q_3} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \underline{J(q)\dot{q}}$

2-2)

$${}^1\dot{p} = \underbrace{\begin{bmatrix} -(l_1+q_2)\sin q_1 - l_2\sin(q_1+q_3) & \cos q_1 & -l_2\sin(q_1+q_3) \\ (l_1+q_2)\cos q_1 + l_2\sin(q_1+q_3) & \sin q_1 & l_2\cos(q_1+q_3) \\ 1 & 0 & 1 \end{bmatrix}}_{\rightarrow J(q)} \dot{q}$$

2-3) $\tau = J_w^T(q) m_1 = \begin{bmatrix} -(l_1+q_2)\sin q_1 - l_2\sin(q_1+q_3) & (l_1+q_2)\cos q_1 + l_2\cos(q_1+q_3) & 1 \\ \cos q_1 & \sin q_1 & 0 \\ -l_2\sin(q_1+q_3) & l_2\cos(q_1+q_3) & 1 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ n \end{bmatrix}$

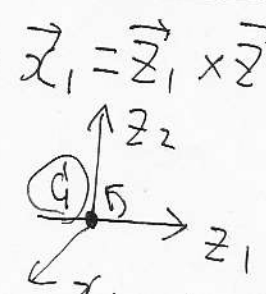
($J_w = J$ for this case planer robot)

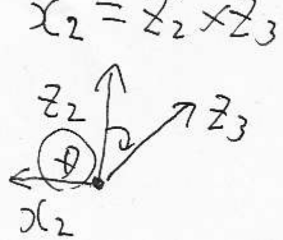
the holding torque (force) is $-\tau = \begin{bmatrix} \{(l_1+q_2)\sin q_1 + l_2\sin(q_1+q_3)\} f_x - \{(l_1+q_2)\cos q_1 + l_2\cos(q_1+q_3)\} f_y - n \\ -\cos q_1 f_x - \sin q_1 f_y \\ l_2\sin(q_1+q_3) f_x - l_2\cos(q_1+q_3) f_y - n \end{bmatrix}$

[3] Note that $\underline{\vec{x}_{i-1} = \vec{z}_{i-1} \times \vec{z}_i}$ $\rightarrow \vec{x}_1 = \vec{z}_1 \times \vec{z}_2$, $\vec{x}_2 = \vec{z}_2 \times \vec{z}_3$

in the D-H method ↓

common perpendicular is very important





Nov. 26th, 2019

(65)

Answer Sheet (Robotics Mid-Exam and Robotics I Exam) No. /

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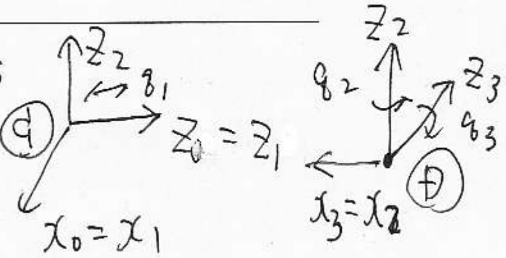
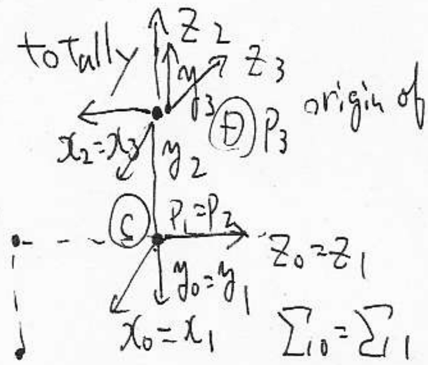
[3] follow $Z_0 \equiv Z_1$, $X_0 \equiv X_1$, $X_3 \equiv X_2$ ⇒ thus



(3-1) totally



origin of Σ_2 and origin of Σ_3 is same (but $\Sigma_2 \neq \Sigma_3$)

(y is automatically defined)

$\Sigma_{10} = \Sigma_1$

(3-2)

$$\begin{cases} P_1 : \text{foot of } X_0 \text{ on } Z_1 = Ⓒ \\ P_2 : \text{foot of } X_1 \text{ on } Z_2 = Ⓒ \\ P_3 : \text{foot of } X_2 \text{ on } Z_3 = Ⓓ \end{cases}$$

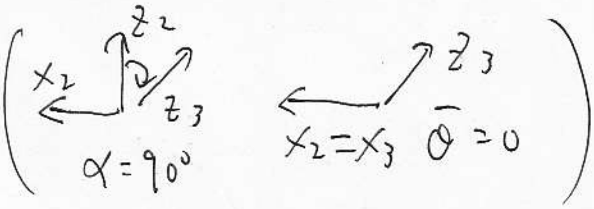
$$\begin{cases} \alpha_1 : Z_1, X_0 = 0 \\ \alpha_2 : Z_2, Z_0, X_1 = 90^\circ \\ \alpha_3 : Z_1, X_2, Z_2 = 90^\circ \end{cases}$$

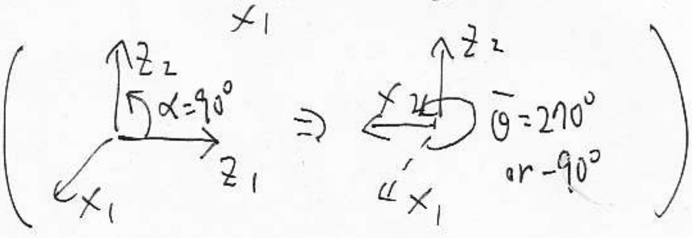
$$\begin{cases} a_1 : \overline{\Sigma_0 P_1} \text{ on } X_0 = 0 \\ a_2 = \overline{\Sigma_1 P_2} \text{ on } X_1 = 0 \\ a_3 = \overline{\Sigma_2 P_3} \text{ on } X_2 = 0 \end{cases}$$

$$\begin{cases} d_1 : \overline{P_1 \Sigma_1} = 0 + \underline{\underline{q_1}} \\ d_2 : \overline{P_2 \Sigma_2} = \underline{\underline{l_2}} \\ d_3 : \overline{P_3 \Sigma_3} = 0 \end{cases}$$

$$\begin{cases} \theta_1 : X_1, Z_1, X_0 = 0 \\ \theta_2 : X_2, Z_2, X_1 = 270^\circ + \underline{\underline{q_2}} \end{cases}$$

$$\theta_3 = X_3, Z_3, X_2 = 0 + \underline{\underline{q_3}}$$





i	a_i	α_i	d_i	θ_i
1	0	0	q_1	0
2	0	90°	l_2	$270 + q_2$
3	0	90°	0	q_3

Totally

i	1	2	3
a_i	0	0	0
α_i	0	90°	90°
d_i	q_1	l_2	0
θ_i	0	$q_2 + 270^\circ$ (or -90°)	q_3

⇓

in each i, q_i must be included in the D-H table!

(3-3) $${}^0{}^1P_H = {}^0T_3\, {}^3{}^1P_H = {}^0T_3 \begin{bmatrix} -l_H \\ 0 \\ 0 \\ 1 \end{bmatrix}$$