クラス　　番号　　氏名

[1] $\ddot{z} - \alpha z = 0$ で $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$ として $\dot{x} = Ax$ を示せ

$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = x_2$
$= \alpha z = \alpha x_1$

$\Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix} x$
↳ A

[2] この A で固有値固有ベクトルを求めよ

$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ \alpha & -\lambda \end{vmatrix} = \lambda^2 - \alpha = 0 \rightarrow \lambda = \pm\sqrt{\alpha}$ (2つの固有値)

(i) $\lambda = \sqrt{\alpha}$ のとき $Av = \lambda v \rightarrow A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ \alpha x \end{bmatrix} = \sqrt{\alpha}\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow v_1 = \begin{bmatrix} 1 \\ \sqrt{\alpha} \end{bmatrix} t$ (なくても可)

(ii) $\lambda_2 = -\sqrt{\alpha}$ のとき

[3] この A で $e^{At} = V\begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}V^{-1}$ $= \begin{bmatrix} y \\ \alpha x \end{bmatrix} = -\sqrt{\alpha}\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow v_2 = \begin{bmatrix} 1 \\ -\sqrt{\alpha} \end{bmatrix} t$

t は任意定数（0でない）

t_2 として

により計算せよ

$V = \begin{bmatrix} 1 & 1 \\ \sqrt{\alpha} & -\sqrt{\alpha} \end{bmatrix}$, $V^{-1} = \frac{1}{-2\sqrt{\alpha}}\begin{bmatrix} -\sqrt{\alpha} & -1 \\ -\sqrt{\alpha} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2\sqrt{\alpha}} \\ \frac{1}{2} & -\frac{1}{2\sqrt{\alpha}} \end{bmatrix}$

よって $e^{At} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ \sqrt{\alpha} & -\sqrt{\alpha} \end{bmatrix}\begin{bmatrix} e^{\sqrt{\alpha}t} & 0 \\ 0 & e^{-\sqrt{\alpha}t} \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{\sqrt{\alpha}} \\ 1 & -\frac{1}{\sqrt{\alpha}} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} e^{\sqrt{\alpha}t} + e^{-\sqrt{\alpha}t} & \frac{1}{\sqrt{\alpha}}(e^{\sqrt{\alpha}t} - e^{-\sqrt{\alpha}t}) \\ \sqrt{\alpha}(e^{\sqrt{\alpha}t} - e^{-\sqrt{\alpha}t}) & e^{\sqrt{\alpha}t} + e^{-\sqrt{\alpha}t} \end{bmatrix}$

[4] $e^{At} = \mathcal{L}^{-1}[sI - A]^{-1}$ により計算せよ

$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ -\alpha & s \end{bmatrix}^{-1} = \frac{1}{s^2 - \alpha}\begin{bmatrix} s & 1 \\ \alpha & s \end{bmatrix} = \begin{bmatrix} \frac{A_1}{s+\sqrt{\alpha}} + \frac{B_1}{s-\sqrt{\alpha}} & \frac{A_2}{s+\sqrt{\alpha}} + \frac{B_2}{s-\sqrt{\alpha}} \\ \frac{A_3}{s+\sqrt{\alpha}} + \frac{B_3}{s-\sqrt{\alpha}} & \frac{A_1}{s+\sqrt{\alpha}} + \frac{B_1}{s-\sqrt{\alpha}} \end{bmatrix}$ → 4

↳ $\frac{1}{(s+\sqrt{\alpha})(s-\sqrt{\alpha})}$

$A_1 = \left[\frac{s}{s+\sqrt{\alpha}}\right]_{s=-\sqrt{\alpha}} = \frac{-\sqrt{\alpha}}{-2\sqrt{\alpha}} = \frac{1}{2}$, $B_1 = \frac{1}{2}$, $A_2 = \left[\frac{1}{s-\sqrt{\alpha}}\right]_{s=-\sqrt{\alpha}} = -\frac{1}{2\sqrt{\alpha}}$, $B_2 = \frac{1}{2\sqrt{\alpha}}$, $A_3 = \left[\frac{\alpha}{s-\sqrt{\alpha}}\right]_{s=-\sqrt{\alpha}} = -\frac{1}{2}\sqrt{\alpha}$, $B_3 = \frac{1}{2}\sqrt{\alpha}$

4 → $\mathcal{L}^{-1}\ \frac{1}{2}\begin{bmatrix} e^{\sqrt{\alpha}t} + e^{-\sqrt{\alpha}t} & \frac{1}{\sqrt{\alpha}}(e^{\sqrt{\alpha}t} - e^{-\sqrt{\alpha}t}) \\ \sqrt{\alpha}(e^{\sqrt{\alpha}t} - e^{-\sqrt{\alpha}t}) & e^{\sqrt{\alpha}t} + e^{-\sqrt{\alpha}t} \end{bmatrix} = e^{At}$

([3]と同じ)